Gabriel Durazo
Compressible Flow MMAE 311
November 10th, 2026

Nozzle Analyzer

1. Problem Definition and Assumptions

The project analyzes one SSME-shaped nozzle at two chamber operating points. Part 1 computes the design nozzle flow distributions and the vacuum scenario. Part 2 compares an ideal isentropic situation with a flow from the given Pitot measurement.

The model assumes steady, quasi 1D, axisymmetric flow of a calorically perfect gas. Each smooth region is isentropic; the non-isentropic model contains one adiabatic normal shock. This model excludes boundary layers, separation, chemistry, heat transfer, and multidimensional shock structure.

Input	Part 1: design point	Part 2: off-design
Chamber stagnation pressure	20.4 MPa	10 atm = 1.01325 MPa
Stagnation temperature	3500 K	2000 K
Assigned throat area	0.30 m^2	0.30 m^2
Exit area used	23.253 m^2 (contour)	23.256 m^2 (assigned)
Additional pressure input	Design static p_e=18.4592 kPa	Pitot stagnation pressure=0.5 atm

Common properties: γ = 1.2; MW = 22 kg/kmol; R=R_u/MW = 377.930 J/(kg K); c_p = γR/(γ-1) = 2267.581 K/(kg K)l g_0 = 9.80665 m/s^2. All calculations use SI units. Sea level is 101.325 kPa; the formal vacuum pressure is zero

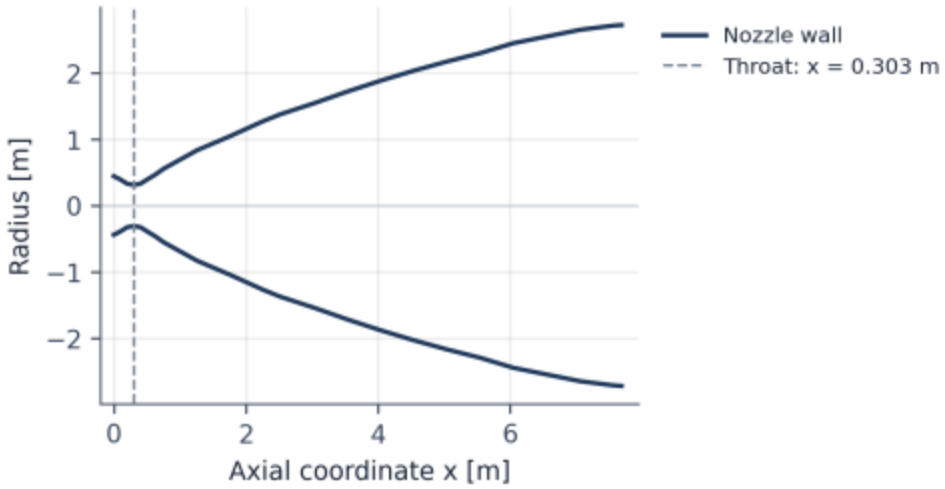


Figure 1. Assigned nozzle contour, converted from centimeters to meters. The minimum-radius station is at x = 0.303 m. The coordinate origin is the first supplied station.

Using the contour values, the radius was seen to be about 0.30 m^2. The Jupyter notebook uses that throat area and sets M = 1 at that point. There was a small difference between the given radius and the calculated radius, which is mentioned in the code

2. Numerical Approach and Program Structure

The Jupyter notebook starts off by laying down constants and the nozzle shape. It then sets up residuals and derivative functions that will be used for iteration to solve Part 1's design nozzle. It then reuses those functions for Part 2 and checks the work. NumPy is used with these iterations

2.1 Area-Mach solver

$$\Phi(m) = \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{A}{A^*}$$

$$F(M) = \Phi(M) - AR, \qquad M_{j+1} = M_j - \frac{F(M_j)}{F'(M_j)}$$

The analytical derivative is evaluated using the substitutions H and n:

$$H = \frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right), \qquad n = \frac{\gamma+1}{2(\gamma-1)}, \qquad H' = \frac{2(\gamma-1)}{\gamma+1}M,$$

$$F'(M) = \frac{nH^{n-1}H'}{M} - \frac{H^n}{M^2}$$

For A/A* > 1, the subsonic solve starts at M = 0.01 and the supersonic solve starts at M = 3.0. There is a midpoint correction that replaces the new steps that take off from the wanted branch, subsonic or supersonic. At A/A* = 1, the function returns back to M = 1 since the derivative disappears there

The program stops the area-Mach solve when $|M_{j+1}-M_j|$ becomes less than 1/100, with 100 iterations being the max it will go. Part 1 selects the subsonic branch before the throat, M =1 at the throat, and the supersonic branch after the throat. The program then uses the subsonic branch to solve the post-shock exit

2.2 Normal-shock pressure-ratio solver

Define $Q(M_1)$ as the downstream/upstream stagnation-pressure ratio for a normal shock:

$$Q(M_1) = \left[\frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2+2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$G(M_1) = Q(M_1) - \frac{P_{02}}{P_{01}}$$

$$G'(M_1) = -\,Q(M_1)\frac{4\gamma(M_1^2-1)^{\frac{1}{\gamma-1}}}{M_1\left[(\gamma-1)M_1^2+2\right]\left[2\gamma M_1^2-(\gamma-1)\right]}$$

The program then starts Newton's method with M_1 starting at 4.0 and tries to stay above Mach 1 using the midpoint corrections implemented previously. Convergence occurs once |G| is less than 1/100, with 100 iterations being the max (similar to the area-Mach solver). The tolerance is a pressure-ratio residual, different from the area-Mach tolerance, where it was just a change in Mach number. The unknown is the Mach number right before the internal shock

3. Part 1: Design Point Results

The given design pressure can be seen to have an isentropic exit Mach of 4.70681. This means there is a choked branch throughthe converging diverging section. Our static equations are gathered from the stagnation realtions

$$T = \frac{T_0}{1+\frac{(\gamma-1)M^2}{2}}, \qquad P = \frac{P_0}{\left(1+\frac{(\gamma-1)M^2}{2}\right)^{\frac{\gamma}{\gamma-1}}}$$

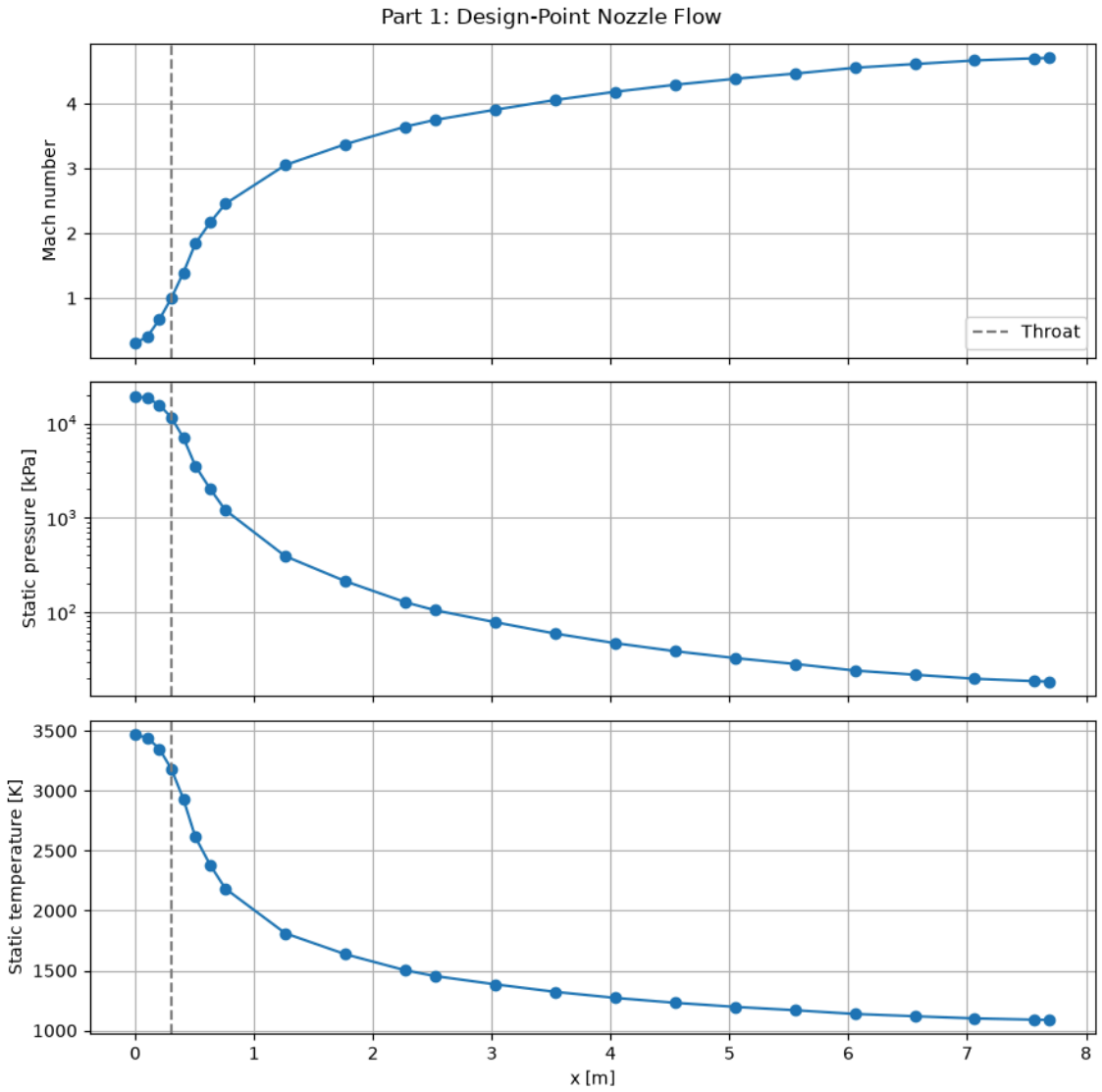


Figure 2. Part 1 distributions at the supplied stations. The dashed line marks the throat. Pressure uses a logarithmic vertical axis to show the large decrease while preserving the positive exit pressure.

Part 1 Results	Value	Part 1 Results	Value

Exit Mach	4.706723	Exit Temperature	1088.54 K
Exit Pressure	18.4621 kPa	Exit Velocity	3307.02 m/s
Mass Flow	3450.98 kg/s	Momentum Thrust	11412.46 kN
Vacuum Pressure Thrust	429.30 kN	Total Vacuum Thrust	11841.76 kN
Vacuum Specific Impulse	349.91	Design Pressure Difference	+0.01576%

$$\dot{m} = \frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2\gamma(\gamma-1)}}, \qquad V_e = M_e \sqrt{\gamma R T_e}$$

Mach increases through the throat and static pressure and temperature decrease. The exit pressure computed was slightly different from the given value by about 0.016%, which is consistent with the small error found in the throat radius. Thrust includes both the momentum and exit pressure

4. Part 2: Pitot Diagnosis and Shock Reconstruction

At P_{01} = 10 atm and T_0 = 2000 K, the ideal exit Mach is 4.7. At P_{01} = 10 atm and T_0 = 2000 K, the ideal exit Mach is 4.7. A Pitot probe predicts the ideal exhaust would be $P_{Pitot} = P_{01} Q(M_e)$ = 0.234 atm before the normal shock. Meaning the ideal exhaust and probe shock does match with the measurement.

The reconstruction tests one internal normal shock followed by isentropic subsonic flow. Under this assumption, the Pitot reading equals the downstream stagnation pressure, so P_{02}/P_{01} = 0.05. Solving G = 0 gives the pre-shock Mach below. This is a model-based inference from the measurement.

$$A_s = A_t \phi(M_1), r_s = \sqrt{\frac{A_s}{\pi}} \qquad M_2 = \sqrt{\frac{1+(\gamma-1)\frac{M_1^2}{2}}{\gamma M_1^2 - \frac{\gamma-1}{2}}}$$

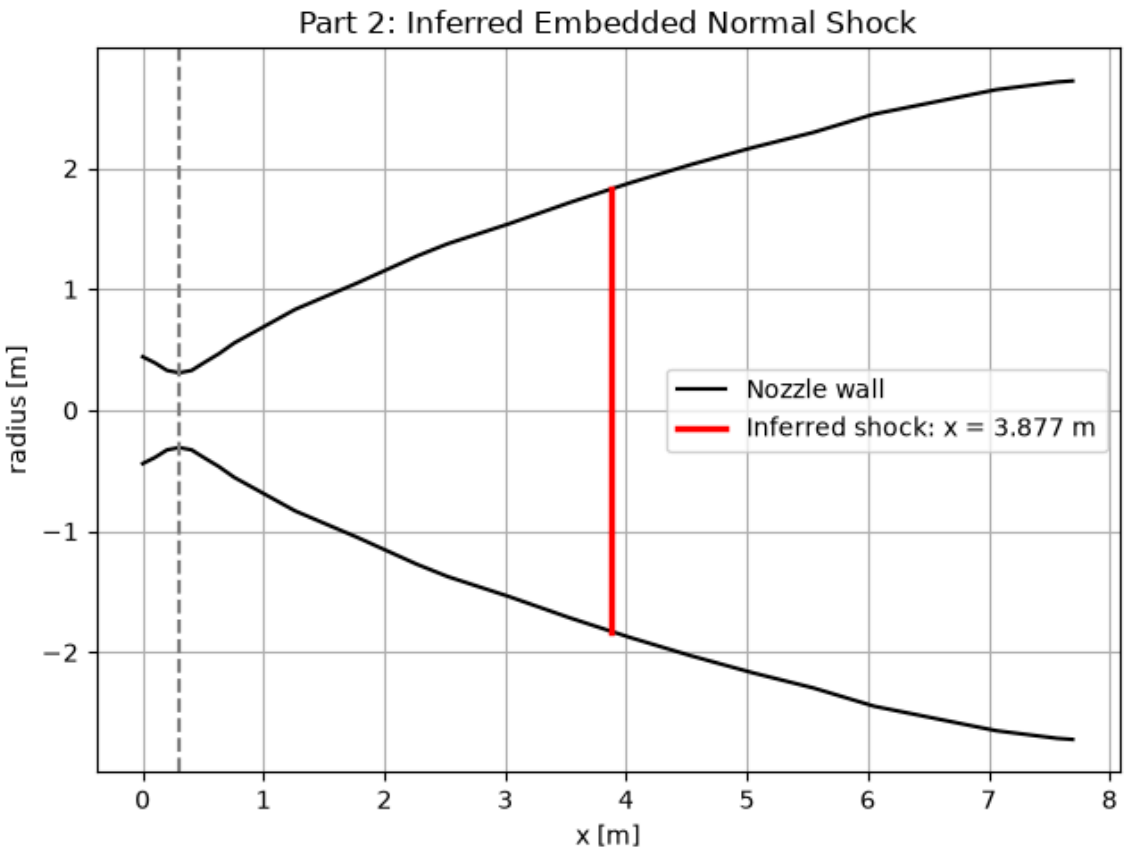


Figure 3. Inferred embedded normal shock. Linear interpolation of radius between the tabulated stations x = 3.532 m and x = 4.036 m gives x_s =3.8768 m, measured from the original contour origin.

Shock / Downstream Quantity	Value	Shock / Downstream Quantity	Value
Mach Before Shock	4.15	Mach After Shock	0.364
Shock Area	10.49 m^2	Shock Radius	1.827 m
Shock x-coordinate	3.88 m	Distance After Throat	3.58 m
Downstream P_{02}	50.6625 kPa	Downstream T_{02}	2000 K
Downstream A_2^*	6 m^2	Exit A_e/A_2^*	3.876
Exit Mach	0.154764	Exit Velocity	147.22 m/s
Exit Pressure	49.94 kPa	Exit Temperature	1995.22 K

$$P_{01}A_1^* = P_{02}A_2^*, \quad A_2^* = A_t \frac{P_{01}}{P_{02}} = 6.0\,m^2$$

Mass flow and stagnation temperature are conserved across the adiabatic shock. The reduced stagnation pressure therefore requires a larger downstream reference sonic area. A_2^* is used in the downstream area-Mach relation; l the physical geometry remains the assigned contour. Applying the subsonic exit root and downstream stagnation conditions gives the exit state above.

5. Part 2: Completed Performance Comparison

The requested thrust and specific-impulse calculations use the standard relations:

$$F = \dot{m}V_e + (P_e - P_\infty)A_e, \qquad I_{sp} = \frac{F}{\dot{m}g_0}$$

Table interpretation: each model retains its computed mass flow and exit state while the ambient pressure in the thrust equation is set to sea level or vacuum. The shock reconstruction retains the same inferred shock location in both columns. These are format comparisons, not four independently solved physical operating states.

Quantity	Isentropic Sea Level	Shock Model Sea Level	Isentropic Vacuum	Shock Model Vacuum
Ambient Pressure [kPa]	101.325	101.325	0	0
Mass Flow [kg/s]	226.75	226.75	226.75	226.75
Shock x-coordinate [m]	-	3.88	-	3.88
Mach Before Internal Shock	-	4.15	-	4.15
Exit Mach	4.706815	0.154764	4.706815	0.154764
Exit Static Pressure [kPa]	0.916849	49.9405	0.916849	49.9405
Exit Temperature [K]	622.00	1995.22	622.00	1995.22
Exiot Velocity [m/s]	2499.89	157.22	2499.89	147.22
Momentum Thrust [kN]	566.85	33.38	566.85	33.38
Pressure Thrust [kN]	-2335.09	-1195.00	21.32	1161.42
Formal Total Thrust [kN]	-1768.24	-1161.62	588.17	1194.80
Formal Specific Impulse [s]	-795.19	-522.39	264.51	537.31

Table 1. Completed Part 2 summary, with additional exit temperature, exit velocity, and pressure-thrust rows. Shock position uses the original contour origin. A dashed line indicates that the ideal isentropic model has no embedded shock

5.1 Worked Substitution: Shock-Containing State

The momentum contribution is (226.75 kg/s)(147.2 m/s) = 33.38 kN. In the formal vacuum calculator, the exit-pressure term is (49940 Pa)(23.256 m^2) = 1161.42 kN.
Their sum is 1194.80 kN. Dividing by (226.75 kg/s)(9.81 m/s^2) gives the formal value I_{sp} = 537.31 s. Changing only the ambient pressure to 101325 Pa subtracts $P_\infty A_e$ = 2356.41 kN from this fixed-state result.

5.2 Immediate Comparison

The inferred shock reduces V_e from 2499.89 to 147.22 m/s and reduces momentum thrust by 94.11%. The larger formal vacuum total results from the pressure contributions. Its physical meaning depends on the boundary condition check discussed next.

6. Interpretation, Checks, and Conclusions

6.1 What Is Happening Here?

A normal shock increases entropy and reduces stagnation pressure while conserving T_0. In this reconstruction, only 5% of the upstream stagnation pressure remains. The flow becomes subsonic right after the shock and slows down along the diverging portion. Its T_e ends up approaching T_0.

The ambient pressure comparison is very sensitive. A steady 1D subsonic exit must match the receiving back pressure. The reconstructed P_e = 49.9 kPa implies a back pressure of about 0.493 atm. Keeping that same internal state while imposing either zero or 1 atm at the boundary is inconsistent with this condition. Back pressure changes the shock position; sufficiently low back pressure removes the shock outside the nozzle.

Therefore, the apparently higher shock-model vacuum I_{sp} does not establish an efficiency improvement, and the negative sea-level values do not establish actual operating thrust. They are results of the stated fixed-exit-state substitutions. Physically consistent prediction for each ambient condition would require solving the internal flow again using that back pressure; those additional operating-point solves are not performed by this notebook.

6.2 Numerical Verification

Check	Result
Sonic area-Mach identity	$\phi(1) = 1$
Largest Part 1 area-ratio residual, excluding rounded throat	1.14e-13
Recovered internal-shock pressure ratio	0.05 (target 0.05)
Absolute shock pressure-ratio residual	6.94e-17
Part 2 throat / reconstructed exit mass flow	226.8 / 226.8 kg/s

The throat and exit mass flows agree to numerical roundoff. The downstream root also satisfies $M_e < M_2 < 1$, consistent with the selected subsonic branch. All 39 code cells in the supplied notebook execute sequentially in a clean Python state.

6.3 Findings

The design point solution gives vacuum thrust 11.84 MN and I_{sp} = 349.91 s. The Part 2 Pitot reading is consistent, under the single embedded-shock hypothesis, with M_1 = 4.15 and x_s = 3.88 m. The reconstructed exit and mass-conservation checks support the numerical implementation; ambient-condition consistency limits how its formal performance comparisons can be interpreted